Background

Having collections of data: Good.

Knowing the connections between collections: Better!

Example(s):
Relations (1 / 2)

Definition: (Binary) Relation

Example(s):

Relations (2 / 2)

Definition: Related

Example(s):
Example #1: Presidents–Parties

Recall: 
\[ A = \{ \text{Kennedy, Johnson, Nixon, Carter, Reagan} \} \]
\[ B = \{ \text{Dem, Rep} \} \]
\[ R = \{ (\text{Kennedy, Dem}), (\text{Johnson, Dem}), (\text{Nixon, Rep}), (\text{Carter, Dem}), (\text{Reagan, Rep}) \} \]

Example #2: \( x \% y = 0, x \neq y \)

Recall: 
\[ H = \{ 1, 2, 3, 4, 5, 6 \} \]
\[ R = \{ (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (4, 2), (6, 2), (6, 3) \} \]
Properties of Relations: Reflexivity

**Definition:** Reflexivity

**Example(s):**

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Properties of Relations: Symmetry (1 / 2)

**Definition:** Symmetry

**Example(s):**

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Properties of Relations: Symmetry (2 / 2)

Example(s): Graph Representations & Symmetry

Properties of Relations: Antisymmetry (1 / 2)

Definition: Antisymmetry

Example(s):
Properties of Relations: Antisymmetry (2 / 2)

Example(s): Graph Representations & Antisymmetry

Properties of Relations: Transitivity (1 / 2)

Definition: Transitivity

Example(s):
Properties of Relations: Transitivity (2 / 2)

Example(s):

Relational Composition Examples (1 / 4)

Three examples of creating relations from relations.

Example #1: Set Operators
Example #2: Swapping content of ordered pairs

Definition: Inverse

Example #3: Composites

Definition: Composite

Example(s):
Example #3: Composites (cont.)

Example(s):

Definition: Complement

Matrix Representation of Relations (1 / 4)

(Assumption: Relations are on just one set.)

The 0-1 matrix representation of relation $R$ on set $A$ is $|A| \times |A|$, with both dimensions labeled identically. When $(a, b) \in R$, then $\text{matrix}[a][b]=1$. Else, $\text{matrix}[a][b]=0$.

Example(s):
Matrix Representation of Relations (2 / 4)

Observation #1: Detecting Reflexivity

⇒ A relation is reflexive when its corresponding matrix representation has all 1’s along the main diagonal

Example(s):

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Matrix Representation of Relations (3 / 4)

Observation #2: Detecting Symmetry

⇒ Let matrix $M$ represent relation $R$. $R$ is symmetric when $m_{ij} = 1$ iff $m_{ji} = 1$ is true

Example(s):

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Observation #3: Detecting Transitivity

\[ \Rightarrow \text{Let matrix } M \text{ represent relation } R. \text{ } R \text{ is transitive when the non-zero elements of } M^2 \text{ (or of } M^{[2]} \text{) are also non-zero in } M. \]

Example(s):
Equivalence Relations (2 / 4)

Example(s):

So . . . why are these called equivalence relations?

Recall:

\[ R = \{(0,0), (1,1), (1,-1), (-1,1), (-1,-1), (2,2), (2,-2), (-2,2), (-2,-2)\} \]
Equivalence Relations (4 / 4)

Definition: Equivalence Class

Example(s):

Partial Orders (1 / 3)

Consider scheduling the construction of a house.

Definition: Reflexive (a.k.a. Weak) Partial Order
Partial Orders (2 / 3)

Example(s):

Partial Orders (3 / 3)

Definition: Irreflexivity (of Relations)

Definition: Irreflexive (a.k.a. Strict) Partial Order
Total Orders (1 / 2)

**Definition: Comparable**

- ...
- ...
- ...

**Definition: Total Order**

- ...
- ...
- ...

Total Orders (2 / 2)

**Example(s):**

- ...
- ...
- ...

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