Set Concepts Already Covered

You may wish to review these basic set concepts, previously covered in the Math Review appendix, before starting this topic:

- Properties of sets (e.g., duplicate members are not allowed)
- Set notation (membership, set builder notation, etc.)
- Operators (union, intersection, difference, complement, cardinality)
- Venn diagrams
Why Are We Studying Sets?

(Sets are trivial . . . aren’t they?)

Sets are foundational in many areas of Computer Science.

E.g.:

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Subsets

**Definition: Subset**

**Definition: Proper Subset**

**Example(s):**
Set Equality

Definition: Set Equality

Example(s):

Power Sets

Definition: Power Set

Example(s):
Generalized Forms of $\cup$ and $\cap$

Remember summation and product notations? E.g.:

$$\sum_{n=0}^{9} (2n + 1)$$

Similar notation is used to generalize the union and intersection operators.

Assuming that $A_1 \ldots A_m$ and $B_1 \ldots B_n$ are sets, then:

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Two More Set Properties

**Definition: Disjoint**

**Definition: Partition**

**Example(s):**
Examples of Set Identities

Look familiar?

**Associativity**

\[(A \cap B) \cap C = A \cap (B \cap C)\]
\[(A \cup B) \cup C = A \cup (B \cup C)\]

**Commutativity**

\[A \cap B = B \cap A\]
\[A \cup B = B \cup A\]

**Distributivity**

\[A \cap (B \cup C) = (A \cap B) \cup (A \cap C)\]
\[A \cup (B \cap C) = (A \cup B) \cap (A \cup C)\]

**De Morgan**

\[\overline{A \cup B} = \overline{A} \cap \overline{B}\]
\[\overline{A \cap B} = \overline{A} \cup \overline{B}\]

*Note:* As with logical identities, you need not memorize set identities.

Expressing Set Operations in Logic

We’ve seen the first two already.

\[X \subseteq Y \equiv \forall z \ (z \in X \rightarrow z \in Y)\]
\[X \subset Y \equiv \forall z \ (z \in X \rightarrow z \in Y) \land \exists w \ (w \notin X \land w \in Y)\]

For those that return sets, Set Builder notation is a good choice:
To prove that set expressions $S$ and $T$ are equal, we can:

1. Prove that $S \subseteq T$ and $T \subseteq S$, or

2. Convert the equality to logic, prove it, and convert back

**Example(s):**

**Conjecture:** $S \cup U = U$
Conjecture: $S \cup U = U$
Definition: **Ordered Pair**

Example(s):

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Definition: **Cartesian Product**

Example(s):

Notes:
Example: Computer Representation of Sets